## PROBLEM 1

Consider the propagation of transversal electromagnetic waves in a non-magnetized cold plasma. The wave frequency is sufficiently high so that the motion of the ions can be neglected. You can also neglect the plasma pressure term in the momentum-balance equation. Furthermore, the wave amplitude is sufficiently low so that you can use the linear approximation.

a. Derive an expression relating the electric-field amplitude $E_1$ to the current density $j_1$, wave-frequency $\omega$ and wave-number $k$.

b. Use the momentum-balance equation to find a relation between $E_1$ and the first-order electron velocity $u_1$.

c. Derive the dispersion relation $\omega(k)$ for these waves and sketch this relation in a diagram. For which frequencies $\omega$ can electromagnetic waves propagate in the plasma? Give an intuitive explanation.

d. When electromagnetic waves cannot propagate, they are exponentially damped, i.e. $E_1 \sim \exp(-x/\delta)$. Derive an expression for the so-called skin-depth $\delta$.

e. Collisions between the electrons and the background ions can be taken into account by adding a collision term $-\nu_{ei} m_e u_1$ to the right-hand-side of the momentum balance equation. Show that this leads in the dispersion relation to the substitution $m_e \to m_e(1 + i\nu_{ei}/\omega)$.

## PROBLEM 2

Consider high-frequency, electrostatic Langmuir oscillations in a cold, magnetized plasma with the electrons oscillating perpendicularly to the magnetic field. Take the magnetic field in the $z$-direction and the electrons oscillating in the $x$-direction. Neglect the plasma pressure.

a. Use fluid equations to relate the velocity $u_x$ of the electrons to the electric-field amplitude $E$.

b. Derive the dispersion relation for these oscillations.

c. How do the electron trajectories look like?

## PROBLEM 3

A 8 mm microwave interferometer is used on an infinite plane-parallel plasma slab with a thickness of 8 cm.

a. The transmitted microwaves show a phase shift of 1/10 fringe. Calculate the plasma density assuming that the plasma is uniform.

b. Show that if the phase shift is small it is proportional to the plasma density.

## PROBLEM 4

a. Show that the maximum phase velocity of the whistler mode occurs at $\omega = \omega_c/2$ and is smaller than the light velocity $c$.

b. Show that the group velocity of the whistler mode is proportional to $\omega^{1/2}$ if $\omega \ll \omega_c$ and $v_{ph} \ll c$.